CUSV Modeling*

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1 Overview

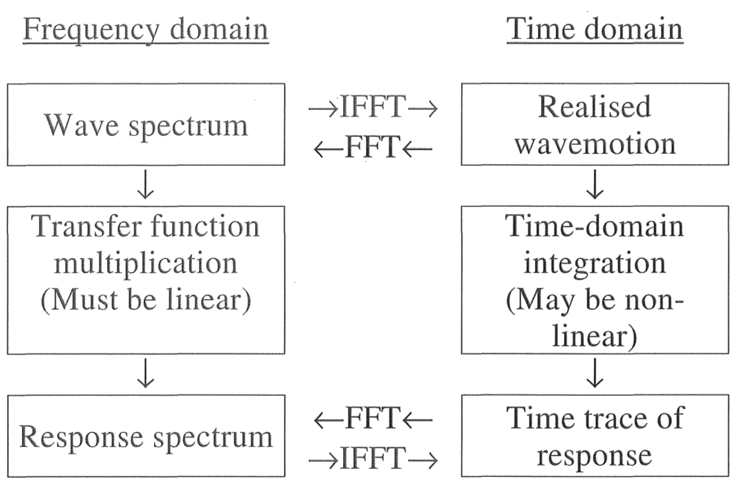

Figure 1: From Bergdahl (2009).

2 Vehicle Model

To approximate the motion of a surface vehicle in the ocean environment, we adapt the six degree-of-freedom robot-like vectorial model for marine craft, proposed by Fossen (2011), and expressed as[1]

$$\underbrace{\boldsymbol{M}_{RB}\dot{\boldsymbol{\nu}} + \boldsymbol{C}_{RB}(\boldsymbol{\nu})\boldsymbol{\nu}}_{\text{rigid-body forces}} + \underbrace{\boldsymbol{M}_{A}\dot{\boldsymbol{\nu}}_r + \boldsymbol{C}_{A}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r}_{\text{hydrodynamic forces}} + \underbrace{\boldsymbol{g}(\boldsymbol{\eta})}_{\text{hydrostatic forces}} = \boldsymbol{\tau}_{propulsion} + \boldsymbol{\tau}_{wind} + \boldsymbol{\tau}_{waves} \tag{1}$$

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[1]The rigid-body terms can be expressed equivalently using either the total vessel velocity or the velocity relative the fluid, see Property 8.1 of Fossen (2011). For our implementation the rigid-body terms and the hydrodynamics terms are solved for separately, and it is more direct to leverage the native physics engine in terms of total rigid-body velocity.

where

$$\boldsymbol{\eta} = [x, y, z, \phi, \theta, \psi]^T \tag{2a}$$

$$\boldsymbol{\nu} = [u, v, w, p, q, r]^T \tag{2b}$$

are the position and velocity vectors respectively for surge, sway, heave, roll, pitch and yaw. The total velocity, $\boldsymbol{\nu}$, is the sum of an irrotational water current velocity, $\boldsymbol{\nu}_c$, and the vessel velocity relative to the fluid, $\boldsymbol{\nu}_r$, i.e., $\boldsymbol{\nu} = \boldsymbol{\nu}_r + \boldsymbol{\nu}_c$. The forces and moments due to propulsion (control), wind, and waves are represented as $\boldsymbol{\tau}_{propulsion}$, $\boldsymbol{\tau}_{wind}$ and $\boldsymbol{\tau}_{waves}$.

Traditionally, surface vessel models are separated into *maneuvering* models (representing the surge, sway and yaw degrees-of-freedom) and *seakeeping* models (representing the heave, pitch and roll degrees-of-freedom). For the purposes of supporting the development of autonomy, it is important that the unified simulation model include *both* maneuvering and seakeeping degrees-of-freedom. The maneuvering aspects of the model influence the vessel steering and control portion of the autonomy solution. The inclusion of the seakeeping aspect of the model is critical for exercising the sensory perception portion of the autonomy solution.

3 Rigid-Body Terms

We assume that the principal axes of inertia can be considered coincident with the body-fixed axes through the center of mass of the vessel. The vessel is symmetric about the $x - z$ plane (left-right symmetry), which implies that $I_{xy} = I_{yx} = 0$. As a deliberate simplification we further treat the vessel as symmetric about the $x - y$ plane (top-bottom symmetry) so that $I_{xz} = I_{yz} = 0$; real hulls differ above and below the waterline, but this idealization yields a diagonal inertia matrix that is adequate for the purposes of this model.

$$\boldsymbol{M}_{RB} = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{xx} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{yy} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{zz} \end{bmatrix}. \tag{3}$$

and a Coriolis-centripetal matrix

$$\boldsymbol{C}_{RB}(\boldsymbol{\nu}) = \begin{bmatrix} 0 & 0 & 0 & 0 & mw & -mv \\ 0 & 0 & 0 & -mw & 0 & mu \\ 0 & 0 & 0 & mv & -mu & 0 \\ 0 & mw & -mv & 0 & I_{zz}r & -I_{yy}q \\ -mw & 0 & mu & -I_{zz}r & 0 & I_{xx}p \\ mv & -mu & 0 & I_{yy}q & -I_{xx}p & 0 \end{bmatrix}. \tag{4}$$

3.1 Parameter Estimation

- Mass is calculated based on an estimate of draft, the effective length and the cross-section of a circle segment. The value is consistent with specifications of similar vessels.
- Moments of inertia are based on an effective cylindrical model

Table 1: Mass properties

Parameter	Value	Units	Description
		Given Values	
L	12.2	m	Length
B	3.34	m	Beam
ρ	1,024	$\mathrm{kg/m^3}$	Density of seawater
		Estimated Values	
T	1.2	m	Draft
		Calculated Values	
A_{WP}	35	$\mathrm{m^2}$	Waterplane area
∇	32	$\mathrm{m^3}$	Displaced volume
m	33,000	kg	Mass
I_{xx}	20,700	$\mathrm{kg \cdot m^2}$	Moment of inertia, roll
$I_{yy} = I_{zz}$	420,000	$\mathrm{kg \cdot m^2}$	Moment of inertia, pitch and yaw

4 Hydrodynamic Terms

The hydrodynamic forces in (1) include the added mass terms due to the inertia of the surrounding fluid and hydrodynamic damping terms due to the vehicle interacting with the surrounding fluid. The terms are captured using coefficients, typically referred to as hydrodynamic derivatives, and expressed using SNAME (1950) notation (Fossen, 2011).

The added mass matrix for our vehicle model is expressed as

$$\boldsymbol{M}_A = -\begin{bmatrix} X_{\dot{u}} & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_{\dot{v}} & 0 & 0 & 0 & Y_{\dot{r}} \\ 0 & 0 & Z_{\dot{w}} & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{\dot{p}} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{\dot{q}} & 0 \\ 0 & N_{\dot{v}} & 0 & 0 & 0 & N_{\dot{r}} \end{bmatrix} \tag{5}$$

where, according to Fossen (2011) we can assume $N_{\dot{v}} = Y_{\dot{r}}$ for a slow moving surface ship.

The surge, sway, yaw components of our model match the maneuvering model used by Sarda et al. (2016). For the heave, roll, pitch components, we follow the six degree-of-freedom coupled motion maneuvering model presented by Fossen (2011) that only contains the diagonal terms for these components. As noted by Fossen (2011), the off-diagonal elements of the added mass matrix, $\boldsymbol{M}_A$, will be small compared to the diagonal elements for most hullforms.

Once the added mass terms that make up $\boldsymbol{M}_A$ are chosen, the derivation by Imlay (1961) provides the Coriolis-centripetal added mass terms that result. The Coriolis-centripetal added mass matrix is expressed using the same hydrodynamic derivatives from $\boldsymbol{M}_A$. The resultant matrix is

$$\boldsymbol{C}_A(\boldsymbol{\nu}_r) = \begin{bmatrix} 0 & 0 & 0 & 0 & -Z_{\dot{w}}w_r & Y_{\dot{v}}v_r + Y_{\dot{r}}r \\ 0 & 0 & 0 & Z_{\dot{w}}w_r & 0 & -X_{\dot{u}}u_r \\ 0 & 0 & 0 & -Y_{\dot{v}}v_r & X_{\dot{u}}u_r & 0 \\ 0 & -Z_{\dot{w}}w_r & Y_{\dot{v}}v_r & 0 & -N_{\dot{r}}r_r & M_{\dot{q}}q_r \\ -Z_{\dot{w}}w_r & 0 & -X_{\dot{u}}u_r & N_{\dot{r}}r_r & 0 & -K_{\dot{p}}p_r \\ -Y_{\dot{v}}v_r - Y_{\dot{r}}r & X_{\dot{u}}u_r & 0 & -M_{\dot{q}}q_r & K_{\dot{p}}p_r & 0 \end{bmatrix}. \tag{6}$$

Since the terms of the maneuvering portion of our unified model are chosen to match those of Sarda et al. (2016), we simply used their theoretical expressions to estimate the values for $X_{\dot{u}}$, $Y_{\dot{v}}$, $N_{\dot{r}}$, and $Y_{\dot{r}}$. To estimate the added mass terms in the seakeeping portion of our model we used the previous work of Greenhow and Ahn (1988) involving the study of added mass and damping of partially submerged horizontal cylinders in heave and sway. From their work we were able to directly estimate the heave added mass, $Z_{\dot{w}}$, by assuming that each pontoon could be represented by a circular cylinder. For the roll and pitch added mass, we assumed that half of this added mass acted at half the beam, for roll, and a quarter of the length, for pitch.

The hydrodynamic damping includes forces due to radiation-induced potential damping, skin friction, wave drift damping, vortex shedding and lifting forces (Fossen, 2011; Krishnamurthy et al., 2005). These effects are aggregated in the hydrodynamic damping matrix

$$\boldsymbol{D}(\boldsymbol{\nu}_r) = \boldsymbol{D}_l + \boldsymbol{D}_n(\boldsymbol{\nu}_r) \tag{7}$$

expressed as a sum of linear and quadratic terms:

$$\boldsymbol{D}_l = -\begin{bmatrix} X_u & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_v & 0 & 0 & 0 & Y_r \\ 0 & 0 & Z_w & 0 & 0 & 0 \\ 0 & 0 & 0 & K_p & 0 & 0 \\ 0 & 0 & 0 & 0 & M_q & 0 \\ 0 & N_v & 0 & 0 & 0 & N_r \end{bmatrix} \tag{8}$$

and

$$\boldsymbol{D}_n(\boldsymbol{\nu}_r) = -\begin{bmatrix} X_{u|u|}|u_r| & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_{v|v|}|v_r| & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & N_{r|r|}|r| \end{bmatrix}. \tag{9}$$

This approximation of the hydrodynamic effects is implemented as a parameterized Gazebo plugin. The user defines the characteristics of the vessel under test through a vessel-specific configuration file that includes the hydrodynamic derivatives. During each time step of the simulation, the plugin is executed with access to the state of the vessel and environment. The hydrodynamic force terms in (1) are calculated based on this state information and the user-defined vessel characteristics. The resulting force and moment values are then applied to the vessel through the Gazebo application programming interface (API) for inclusion in the next iteration of the physics engine.

4.1 Drag Estimates

The resistance R due to any particular drag component is expressed as a drag coefficient C_D where

$$R = \frac{1}{2} C_D \rho S U^2$$

where S is the at-rest hull wetted surface and ρ is the water density. The total ship drag, C_T is expressed as

$$C_T = C_W + (1 + k) C_F$$

where C_W is the wave drag, C_F is the frictional drag, and k is a form factor that expresses the viscous pressure drag as a fraction of the frictional resistance.

Holtrop's method Holtrop and Mennen (1982); Holtrop (1984) provides an estimate of the form factor $(1 + k)$ for normal-form hulls.

Table 2: Hull-Form/Fineness Coefficients

Parameter	Symbol	Value	Units
Given Values			
Length of waterline	L	12.2	m
Beam on waterline	B	3.34	m
Estimated Values			
Draft	T	1.2	m
Calculated Values			
Waterplane area	A_{WP}	35	m^2
Maximum section area	A_X	3.01	m^2
Midship section area	A_M	3.01	m^2
Wetted surface Area	S	51	m^2
Displaced volume	∇	32	m^3
Hull-Form Coefficients			
Prismatic	$C_P = \frac{\nabla}{A_X L}$	0.875	
Volumetric	$C_V = \frac{\nabla}{L^3}$	0.018	
Maximum Section	$C_X = \frac{A_X}{BT}$	0.75	
Midship Section	$C_M = \frac{A_M}{BT}$	0.75	
Block	$C_B = \frac{\nabla}{LBT}$	0.66	
Vertical Prismatic	$C_{PV} = \frac{\nabla}{A_{WP} T}$	0.75	
Waterplane	$C_{WP} = \frac{A_{WP}}{LB}$	0.875	
	B/T	2.8	
	$S/V^{2/3}$	5.1	

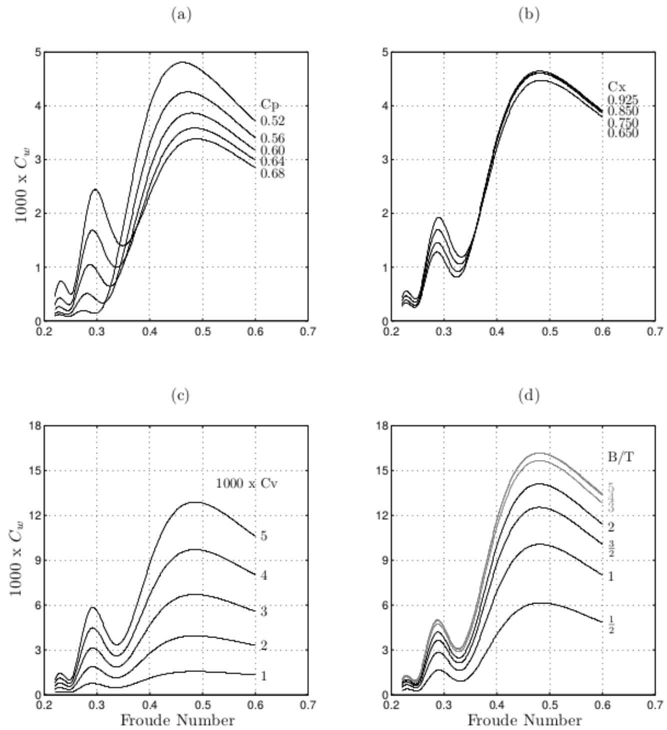

Figure 1.3. Further comparison of linear theory results. Geometric parameters are (a) C_P (b) C_X (c) C_V (d) B/T. Linear theory captures much of the wave drag physics. Compare to Fig. 1.1.

Figure 2: Read (2009)

- Assume a Froude number between $Fr = 0.2 - -0.3$, which is $U = 2.2 - -3.3\,\mathrm{m/s}$.
- Figure 2 suggests $C_W \approx 1.0 \times 10^{-3}$.
- Using Holtrop's method Holtrop and Mennen (1982); Holtrop (1984), we calculate $(1 + k) = 1.76$
- Using the ITTC 1957 friction line $C_F = \frac{0.075}{(\log_{10}(Re) - 2)^2}$ gives $C_F \approx 2.0 \times 10^{-3}$, consistent with the guideline that the majority of the hull's total resistance is due to water friction at slow speeds.

The result is $C_T = 4.5 \times 10^{-3}$, the total resistance force is

$$R = \frac{1}{2} C_T \rho S U^2 = (118.0)\, U^2$$

So

- $X_u = 0$
- $X_{u|u|} = 118.0\,\mathrm{N/(m/s)^2}$
- If similarity to VRX model holds, then
  - $Y_{v|v|} = 11.4 \cdot X_{u|u|} = 1345.0\,\mathrm{N/(m/s)^2}$
  - $Z_{w|w|} = 31.2 \cdot X_{u|u|} = 3680.0\,\mathrm{N/(m/s)^2}$

Table 3: Model Parameters

Parameter	Description	Value	Units	Method
m	Mass	33,000	kg	Displacement estimate based on geometry
I_{xx}	MOI-roll	20,700	$\mathrm{kg \cdot m^2}$	Effective cylinder
$I_{yy} = I_{zz}$	MOI-pitch/yaw	420,000	$\mathrm{kg \cdot m^2}$	Effective cylinder
$X_{\dot{u}}$	Added mass, surge	0	kg	Neglected
$Y_{\dot{v}}$	Added mass, sway	26,000	kg	Cylinder Greenhow and Ahn (1988)
$Z_{\dot{w}}$	Added mass, heave	31,000	kg	Cylinder Greenhow and Ahn (1988)
$K_{\dot{p}} = M_{\dot{q}} = N_{\dot{r}} = Y_{\dot{r}} = N_{\dot{v}} =$	Added mass, roll/pitch/yaw	0	$\mathrm{kg \cdot m^2}$	Neglected
$X_u = 0$	Drag, linear, surge	0	N/(m/s)	Holtrop and Mennen (1982); Holtrop (1984); Read (2009)
$X_{u\|u\|}$	Drag, quadratic, surge	118	$\mathrm{N/(m/s)^2}$	Holtrop and Mennen (1982); Holtrop (1984); Read (2009)
$Y_v = 0$	Drag, linear, sway	0	N/(m/s)	Proportional to VRX model
$Y_{v\|v\|}$	Drag, quadratic, sway	1345	$\mathrm{N/(m/s)^2}$	Proportional to VRX model
K_p, $K_{p\|p}$, M_q, $M_{q\|q\|}$, N_r, $N_{r\|r\|}$	Drag	Various		SIT

5 Summary

6 Frequency Domain Model

Deduced from time-domain model via step response tests. Identification of second-order transfer function parameters via logarithmic decrement. The form of the second order transfer function is

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{10}$$

Table 4: Frequency domain system parameters

Parameter	Symbol	Value	Units
Heave: Figure 3			
Period	T	2.0	s
Damping ratio	ζ	0.13	n/a
Undamped natural freq.	ω_n	3.19	rad/s
Damped natural freq.	ω_d	3.16	rad/s
Pitch: Figure 4			
Period	T	1.6	s
Damping ratio	ζ	0.14	n/a
Undamped natural freq.	ω_n	4.08	rad/s
Damped natural freq.	ω_d	4.04	rad/s
Roll: Figure 5			
Period	T	0.63	s
Damping ratio	ζ	0.021	n/a
Undamped natural freq.	ω_n	9.92	rad/s
Damped natural freq.	ω_d	9.92	rad/s

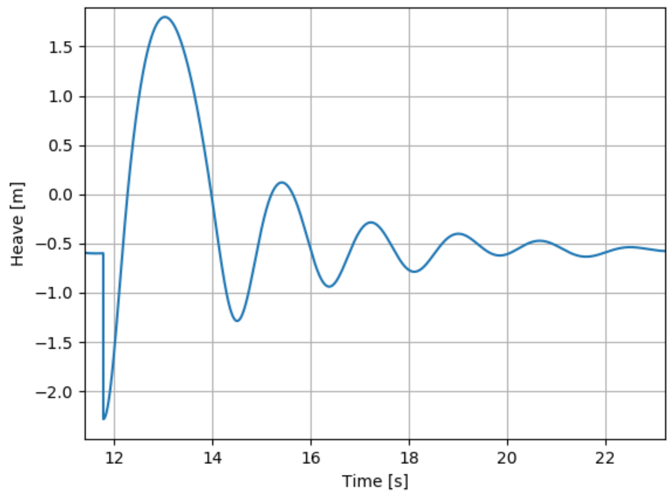

Figure 3: Heave step response

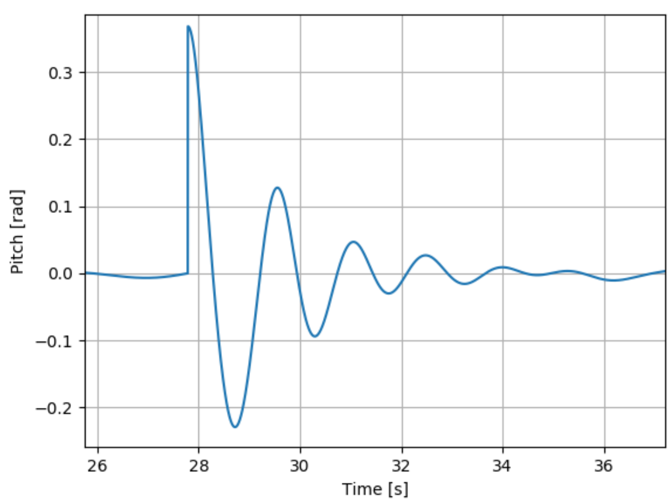

Figure 4: Pitch step response

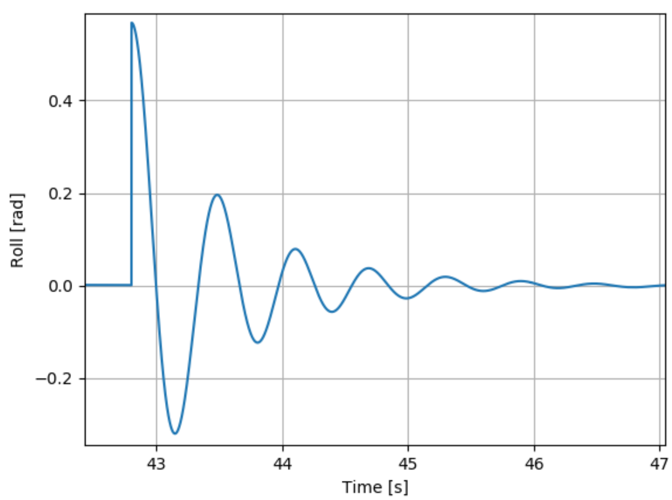

Figure 5: Roll step response

6.1 Spectral Metrics

Based on wave height metrics[2].

$$E[\eta^2(t)] = \sigma_\eta^2 = m_0 = R_{\eta\eta}(\tau = 0) = \int_0^\infty \Gamma_{\eta\eta}(\omega) d\omega \tag{11}$$

where $\eta(x, y, t)$ is the ocean height at a certain location x, y as a function of time t, $E[]$ is the expectation operator, σ_η^2 is the variance in wave height, equivalently the zeroth moment (m_0) and $\Gamma_{\eta\eta}(\omega)$ is the one-sided wave height spectrum as a function of angular frequency.

6.1.1 Wave Slope Spectra

For deep water waves the dispersion relation $\omega = \omega(k)$ is

$$\omega = \sqrt{g\,k}. \tag{12}$$

Because at any time t we can consider a 2D wave and the slope of that wave as

$$\eta(x) = a\cos(kx) \tag{13a}$$

$$\alpha(x) = \frac{d\eta(x)}{dx} = a\,k\cos(kx). \tag{13b}$$

The wave slope spectrum can be expressed as

$$\Gamma_{\alpha\alpha} = k^2\,\Gamma_{\eta\eta}(\omega) = \frac{\omega^4}{g^2}\,\Gamma_{\eta\eta}(\omega) \tag{14}$$

6.1.2 Characteristic Wave Heights

As summarized in Young (1999) the probability density function of wave heights is typically considered to follow the Rayleigh distribution

$$p(H; 2\sigma_\eta) = \frac{H}{(2\sigma_\eta)^2} e^{-H^2/(2(2\sigma_\eta)^2)} \tag{15}$$

Here H is the wave height and the Rayleigh distribution scale parameter is $\sigma = 2\sigma_\eta$.

6.2 Results

The wave-spectrum and vessel-response figures shown below were generated by a separate Python analysis whose source is not preserved with this release.

6.2.1 Wave Spectra: Wave Height and Slope

6.2.2 Vessel Motion Spectra: Heave, Pitch and Roll

[2] http://www.coastalwiki.org/wiki/Statistical_description_of_wave_parameters

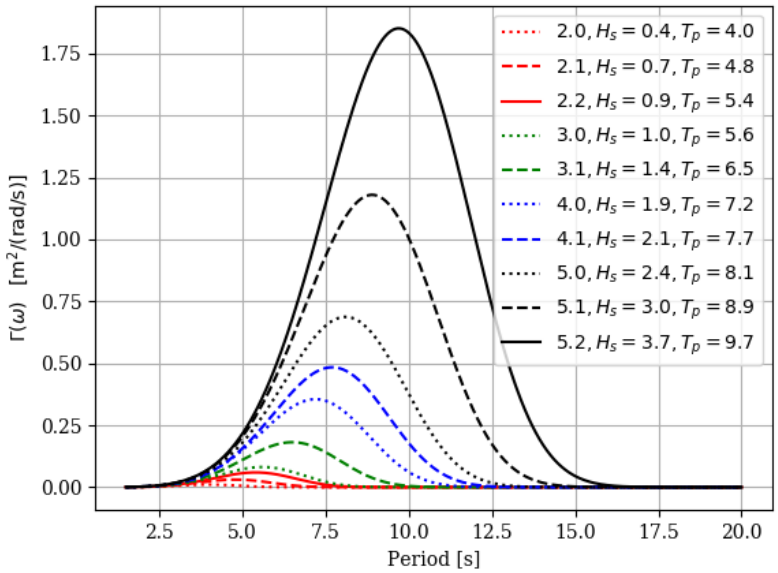

Figure 6: Wave height spectra vs. wave period.

Table 5: Characteristic wave height metrics or representative measures of wave height

Characteristic	Parameter Value	Description
Mean	$\bar{H} = H_{1/1} = \sqrt{2\pi}\,\sigma_\eta = 2.51\,\sigma_\eta$	Average height of all waves.
Root-mean-square	$H_{rms} = 2\sqrt{2}\,\sigma_\eta = 2.83\,\sigma_\eta$	
Significant	$H_{1/3} = H_s = 4\,\sigma_\eta$	Average height of the 1/3rd highest waves.
	$H_{1/10} = 5.09\,\sigma_\eta$	Average height of the 1/10th highest waves.
	$H_{1/100} = 6.67\,\sigma_\eta$	Average height of the 1/100th highest waves.

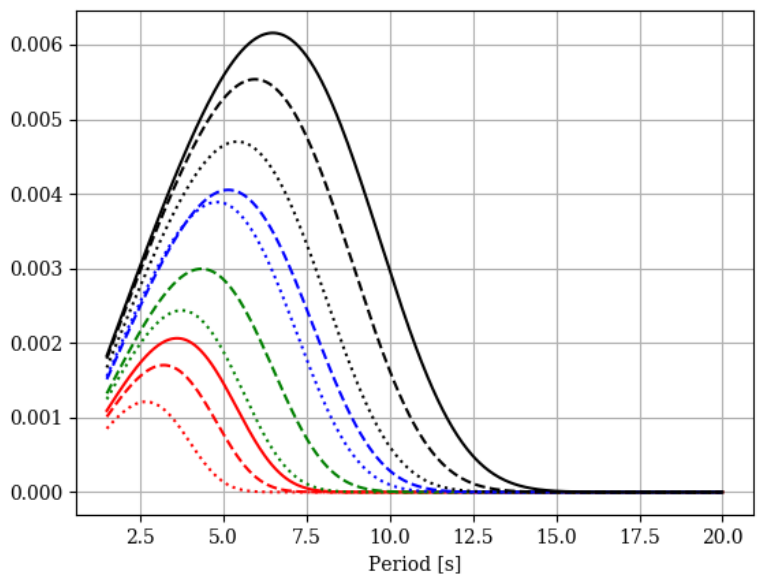

Figure 7: Wave slope spectra vs. wave period.

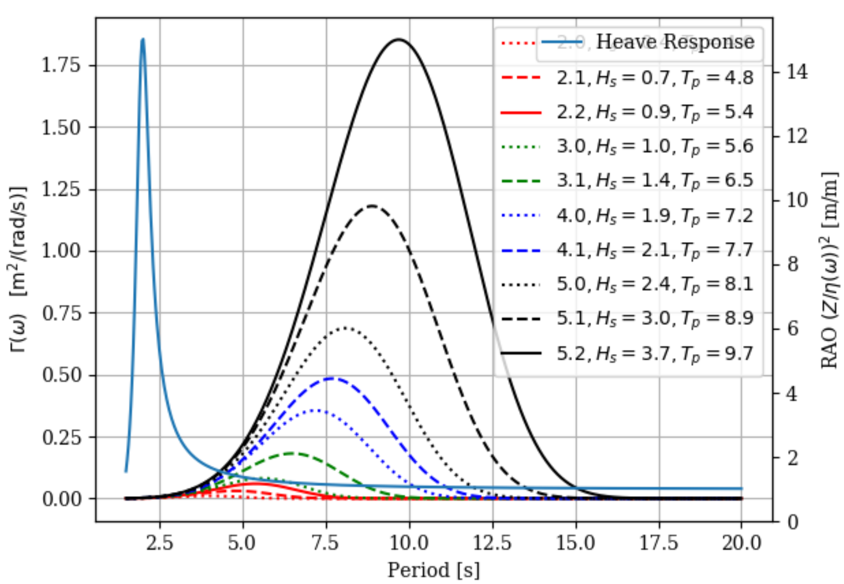

Figure 8: Wave height spectra alongside the heave response as a transfer function.

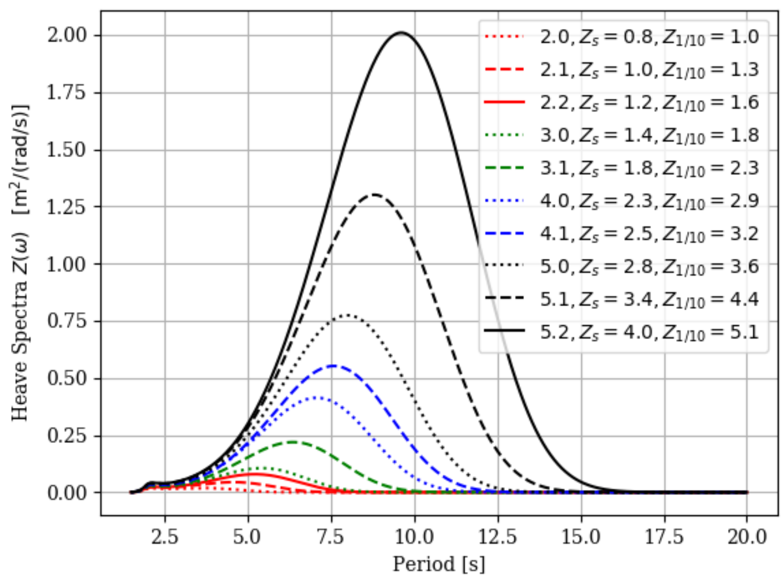

Figure 9: Heave spectra and heave height metrics.

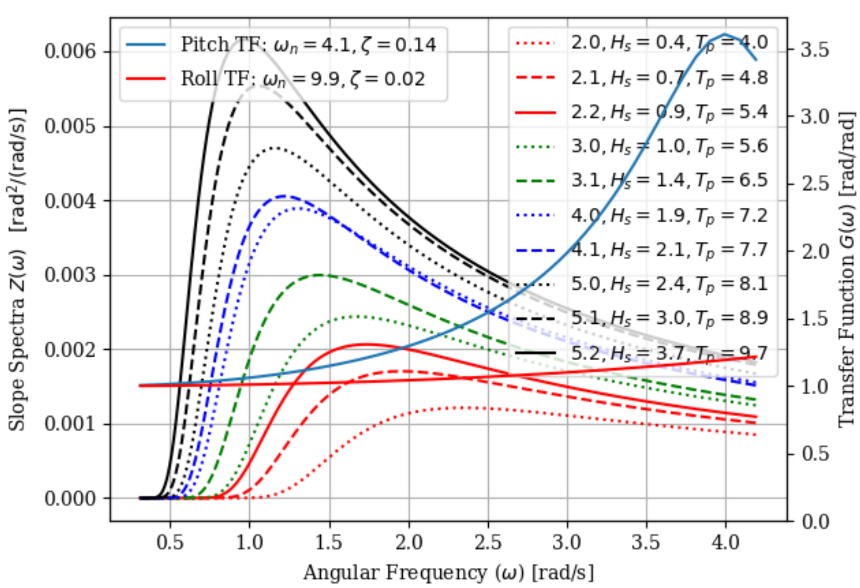

Figure 10: Slope spectra alongside pitch and roll responses as transfer functions.

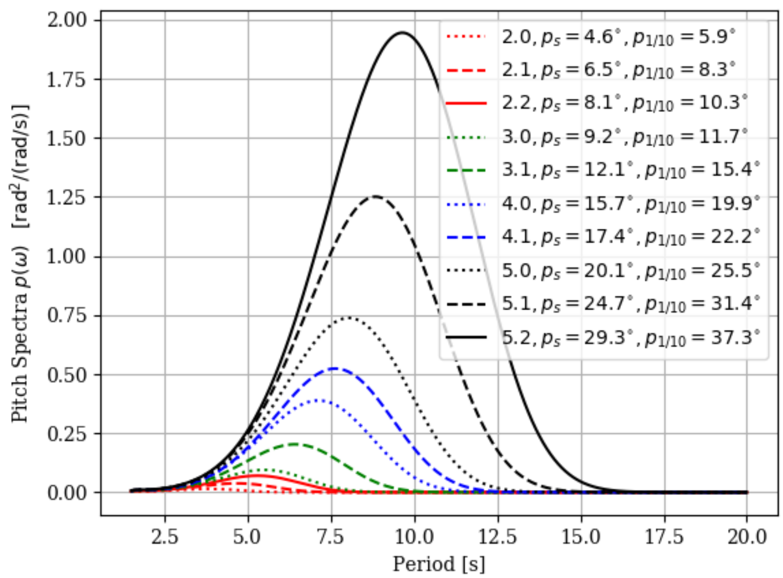

Figure 11: Pitch spectra and metrics, peak-to-peak in degrees

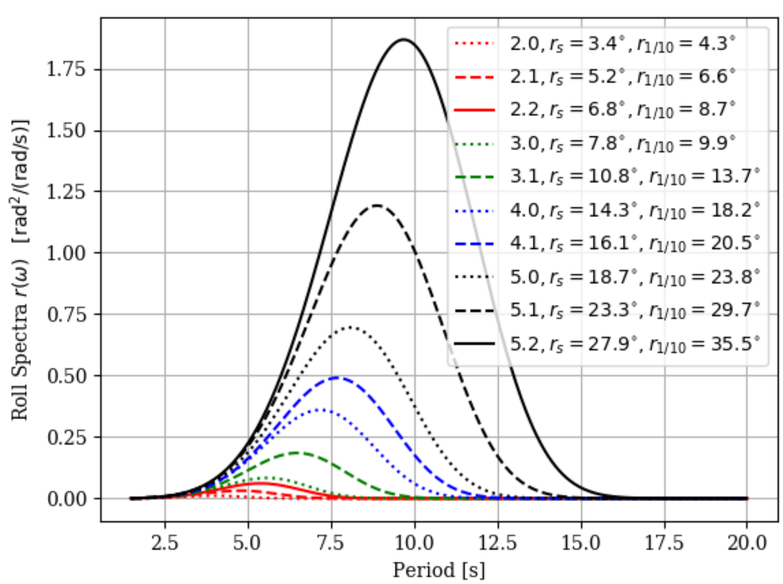

Figure 12: Roll spectra and metrics, peak-to-peak in degrees

Source code

A small set of Python scripts and a spreadsheet supporting the parameter-estimation work above are mirrored alongside this technical note in a `code/` subfolder:

- `area_section_circle.py` — area of a circular segment, used in the mass / displacement estimates (Table 1).
- `haltrop84.py` — Holtrop's method for the form factor $(1 + k)$ used in the drag estimates of Section 4.
- `log_dec.py` — logarithmic-decrement identification of the second-order transfer-function parameters reported in Table 4.
- `usv_param_estimate.ods` — spreadsheet consolidating the parameter calculations.

The current location is `https://github.com/bsb808/carrel/tree/main/posts/usv-control/cusv-model/code`.

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